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3 (Sem-3/CBCS) MAT HC 3

2023

MATHEMATICS

(Honours Core)

Paper : MAT-HC-3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **all** the questions : $1 \times 10 = 10$

(a) When the origin is shifted to a point on the x -axis without changing the direction of the axes, the equation of the line $2x + 3y - 6 = 0$ takes the form $lx + my = 0$. What is the new origin ?

(b) Find the centre of the ellipse

$$2x^2 + 3y^2 - 4x + 5y + 4 = 0.$$

Contd.

- (c) Find the angle between the lines represented by the equation

$$x^2 + xy - 6y^2 = 0.$$

- (d) Transform the equation $\frac{1}{r} = 1 + \cos\theta$ into cartesian form.

- (e) Find the equation of the tangent to the conic $y^2 - xy - 2x^2 - 5y + x - 6 = 0$ at the point $(1, -1)$.

- (f) Express the non-symmetric form of equation of a line $\frac{y}{p} + \frac{z}{c} = 1, x = 0$ in symmetric form.

- (g) Write down the standard form of equation of a system of coaxial spheres.

- (h) Write down the equation of a cone whose vertex is origin and the guiding curve is $ax^2 + by^2 + cz^2 = 1,$
 $lx + my + nz = p.$

- (i) Define a right circular cylinder.

- (j) Find the equation of the tangent plane to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

at the point (α, β, γ) on it.

2. Answer **all** the questions : $2 \times 5 = 10$

- (a) If $(at^2, 2at)$ is the one end of a focal chord of the parabola $y^2 = 4ax,$ find the other end.

- (b) Show that the equation of the lines through the origin, each of which makes an angle α to the line $y = x$ is

$$x^2 - 2xy \sec 2\alpha + y^2 = 0.$$

- (c) Find the point where the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

meets the plane $x + y + z = 3.$

- (d) Find the equation of the sphere passing the points

$$(0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c)$$

- (e) Find the equation of the plane which cuts the surface $2x^2 - 3y^2 + 5z^2 = 1$ in a conic whose centre is $(1, 2, 3)$.

3. Answer **any four** questions : $5 \times 4 = 20$

- (a) Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e \cos \theta$ at the point whose vertical angle is α is given by

$$\frac{l}{r} = e \cos \theta + \cos(\theta - \alpha).$$

- (b) Prove that the line $lx + my = n$ is a

normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)}{n^2}.$$

- (c) Find the asymptotes of the hyperbola $2x^2 - 3xy - 2y^2 + 3x + y + 8 = 0$ and derive the equations of the principal axes.

- (d) Prove that the lines

$$\frac{x+5}{3} = \frac{y+4}{1} = \frac{z-7}{-2} \text{ and}$$

$3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$ are coplanar. Find the equation of the plane in which they lie.

- (e) The section of a cone whose guiding

curve is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$

by the plane $x = 0$, is a rectangular hyperbola. Prove that the locus of the

vertex is $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$.

- (f) Find the centre and the radius of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0,$$

$$x - 2y + 2z = 3.$$

Answer **either (a) or (b)** from the following questions : $10 \times 4 = 40$

4. (a) (i) Find the point of intersection of the lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

(ii) Find the equation of the polar of the point (2, 3) with respect to the conic $x^2 + 3xy + 4y^2 - 5x + 3 = 0$.

5+5=10

(b) (i) Prove that the straight line $y = mx + c$ touches the parabola $y^2 = 4a(x + a)$ if $c = ma + \frac{a}{m}$.

(ii) Find the asymptotes of the hyperbola $xy + ax + by = 0$.

5+5=10

5. (a) Discuss the nature of the conic represented by

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

and reduce it to canonical form.

(b) (i) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(ii) Show that the semi-latus rectum of a conic is the harmonic mean between the segments of a focal chord.

5+5=10

6. (a) (i) A variable plane makes intercepts on the co-ordinate axes, the sum of whose squares is a constant and is equal to k^2 . Prove that the locus of the foot of the perpendicular from the origin to the plane is

$$(x^2 + y^2 + z^2)^2 (x^{-2} + y^{-2} + z^{-2}) = k^2$$

(ii) Two spheres of radii r_1 and r_2 intersect orthogonally. Prove that the radius of the common circle is

$$\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

5+5=10

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0$, $z + x = 0$, $x + y = 0$, $x + y + z = a$

is $\frac{2a}{\sqrt{6}}$ and that the three lines of

shortest distance intersect at the point

$$x = y = z = -a.$$

7. (a) (i) Define reciprocal cone. Show that the cones $ax^2 + by^2 + cz^2 = 0$ and

$$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0 \text{ are reciprocal.}$$

(ii) Find the equation of the right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9$,

$$x - y + z = 3.$$

$$5+5=10$$

(b) (i) Find the equation of the director sphere to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(ii) Show that from any point six normals can be drawn to a conicoid $ax^2 + by^2 + cz^2 = 1$.

$$5+5=10$$